# Remarks on Module2 - Calculus

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A few additional remarks regarding the calculus module[1]:

## Limit Comparison Test of Series

Given a non-negative series and a positive series, $\sum_1^\infty a_n$, $\sum_1^\infty b_n$ with $a_n \geq 0, b_n > 0$, we have an additional tool to verify convergence. The **Limit Comparison Test** states that if the limit

$$0 < \lim_{n\to\infty} \frac{a_n}{b_n} < \infty,$$

i.e. exists, finite and non-zero, then both series converge and diverge together. Meaning, having a positive limit suggests that $\sum_1^\infty a_n$, $\sum_1^\infty b_n$ posses the same behavior at $\infty$.

We use this tool as follows: given a non negative series with terms $a_n$ we try to find an "easy" positive $b_n$ with a positive limit as above. Then we check the convergence of $\sum_1^\infty b_n$ instead of $\sum_1^\infty a_n$

## Surface Area of Revolution

Given a positive function $y = f(x)$, we calculate the volume of the solid revolution revolved around the $x$-axis by:

$$\int_a^b \pi f^2(x)\, dx,$$

as if we are adding together the **area** of disks of radius $f(x)$. If instead, we add together the **surface area** of those disks we get the **Surface Area of Revolution**:

$$\int_a^b 2\pi f(x)\sqrt{1 + f'(x)^2}\, dx.$$

Remember that in both cases we can also revolve the function around the $y$-axis as explained in the lectures. Notice the similarity to the integral calculating an arc's length.

[1]Notations are as in the lectures

## Linear Approximation

Given a curve on the plane $f(x, y) = 0$, the tangent line at point $p = (a, b)$ is

$$f_x|_p(x - a) + f_y|_p(y - b) = 0.$$

When we have an explicit form $y = f(x)$, the tangent line equation is the same as the linear part of the Taylor Expansion at $p$:

$$y = b + f'(a)(x - a).$$

Those concepts are generalized to tangent lines and planes in the Multivariable Calculus module.